Mathematical modelling and field trials of
an inexpensive endoskeletal above-knee prosthesis

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Abstract
The swing-phase motion of the shank of an above-knee prosthesis has been modelled mathematically. An inexpensive endoskeletal prosthesis was designed using the Jaipur foot and conduit pipes with a hinge joint for the knee. Results of field trials and the modelling indicate that a very simple above-knee prosthesis can give near normal gait at "normal" walking speeds on flat surfaces. The swing of the shank is most sensitive to the timing of toe-off.

Introduction
The conventional above-knee (AK) prosthesis poses two major problems for amputees in many low income countries. The usual pelvic suspension system, with its metallic hinge joint at the hip, does not permit abduction and external rotation which is essential for cross-legged sitting on the floor. The rigid thigh and leg pieces of the exoskeletal limb strike against each other when attempting full knee flexion and so the amputee cannot squat. In normal squatting full knee flexion is made possible because the soft tissues of the thigh and knee can be pressed and flattened against each other. Since a Western amputee does not usually assume this posture, the knee joints of most prostheses do not cater for full knee flexion.

The Jaipur AK limb was therefore designed to permit these postures so essential to the Indian life style where floor sitting is a social norm. The rigid metallic hip joint was replaced by flexible leather straps for suspension, allowing freedom of movement in all directions; a posterior elastic strap was strategically located to tighten when sitting cross-legged, thereby keeping the socket pulled up snugly against the stump.

To permit squatting, the endoskeletal concept was utilised. The soft foam covering of the prosthesis can flatten when pressed during full-knee flexion. A new design of knee joint was designed. It not only allowed for full knee flexion but also had an offset hinge to provide alignment stability during the stance phase of walking. An additional locking system was provided as a safety measure. It may be emphasised that the endoskeletal concept was not used as a labour saving device but primarily to allow amputees to squat.

Aluminium sheet was used to fabricate classical quadrilateral sockets and conduit steel pipes for load bearing. The offset single-axis knee-joint and the Jaipur foot (Sethi et al., 1978) are fixed to conduit pipe using a shrink fit technique.

A saucer-shaped aluminium alignment disc is fitted at the top and a static alignment is secured by placing this disc below the socket and temporarily brazing the two together at a few points. The patient is then made to walk and adjustments in alignment are made if required. Once a satisfactory fit and alignment

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are obtained, the alignment disc is securely welded to the socket. A foam covering and a special latex treated cloth cover is then used to provide a proper shape to the limb with a waterproof tear-resistant skin which can withstand the rough exposure of a rural environment. A patient fitted with a Jaipur AK Prosthesis is shown in Figure 1.

The theoretical limits of performance of this AK prosthesis were evaluated with the help of a mathematical model for the swing-phase and limbs were fitted to 200 amputees for field evaluation. (This work was done at the Jaipur Centre.)

Mathematical modelling

A number of studies (Maillardet, 1977; Mena et al., 1981; Mochon and McManus, 1980) have shown that an unconstrained pendulum can approximate the swing-phase motion of the shank since muscles do not play a significant role except in the initial and final stages of the swing-phase (Eberhardt et al., 1968; Maillardet, 1977; Mena et al., 1981). Therefore in this study a mathematical model based on pendulum motion was used to simulate the swing-phase of the prosthesis under different conditions. The objective of the study was to determine the optimal location of the centre of gravity of the prosthesis and also its limitations under different conditions of walking. For the purposes of this analysis the use of the Jaipur Foot and mild steel conduit pipes were taken as given (Fig. 2). The nominal mass of the shank and foot was 1.4 kg. It was assumed that extra mass could only be added along the shank of the prosthesis and a limit of 1.5 kg was fixed for this extra mass M so that the total mass would not exceed 3.0 kg.

The shank was assumed to behave like a pendulum free to rotate about the knee joint. It was also assumed that the prosthesis from the knee downwards was a rigid body which moves through space in the swing-phase under the influence of gravity and a specified trajectory of the knee. The trajectory of the knee was obtained from the literature (Radcliffe, 1976) for “normal” gait on level ground at 95 steps per minute at a speed of 1.1 m/s.

It was assumed that the amputee can move the stump like a normal person hence providing a “normal” knee trajectory for the prosthesis. Then the equation of motion (Lagrangian) for the shank in the swing-phase with a frictionless knee-joint can be shown to be

\[ \ddot{\theta} = - \frac{W_r}{I_o} \left[ \dot{Y} \cos \theta + (\dot{Z} + g) \sin \theta \right] \]

Fig. 1. Amputee fitted with Jaipur above-knee prosthesis without its foam covering.

Fig. 2. Lower leg of a simple above-knee endoskeletal prosthesis with single axis knee joint. Mass of leg with \( M=0 \) is 1.4 kg.
where the terms specified are defined as:
\[ g = \text{The acceleration due to gravity} \]
\[ I_o = \text{Moment of inertia of the shank and foot at the centre of the knee joint about hinge axis} \]
\[ W = \text{Total mass (shank + foot + variable mass M)} \]
\[ r = \text{Distance of centre of gravity (of shank and foot) from the centre of the knee joint} \]
\[ y, z = \text{Cartesian co-ordinates, } y\text{-horizontal, } z\text{-vertical} \]
\[ Y, Z = \text{Displacements of the knee along } y \text{ (forward) and } z \text{ (upward) directions} \]
\[ \dot{Y}, \dot{Z} = \text{Accelerations of the knee along } y \text{ and } z \text{ directions} \]
\[ \theta = \text{The flexion-extension angle of the shank with respect to the vertical, positive in anti-clockwise direction} \]
\[ \ddot{\theta} = \text{Angular acceleration of the shank.} \]

Polynomial curves were fitted to the knee displacement data using a least square technique and integration of the above equation was done numerically using the Merson’s form of the 4th order Runge Kutta method. All computations were done on an ICL 2960 computer.

In order to check the sensitivity of shank motion to friction at the knee joint a few calculations were made assuming different values of the coefficient of friction. The effect of friction was found to be negligible for values of coefficient ranging from 0 to 0.4. This is so because the diameter of the pin is very small and so the moment generated by the mass of the limb is much larger than the moment due to friction.

Therefore, for all other analyses in this study the knee joint was assumed to be frictionless. No knee-stop was provided in the model and so the shank was free to assume a hyper-extended position. Each calculation was terminated at the time of expected heel-strike. Shank position was determined by calculating the shank angle with the vertical at different points in time from toe-off to heel-strike.

**Results**

The shank motion was found to be very sensitive to the value of angular velocity and instantaneous values of vertical acceleration of the shank at toe-off.

The motion of the shank was less sensitive to the location of the centre of the mass. Given the constraints of the geometry of the prosthesis and limitations on maximum allowable mass, it was found that only small variations can be made in the swing-phase motion by altering inertial properties alone.

However, these findings indicate that at "normal" walking speeds on level ground the
shank goes through the gait phase owing to ballistic motion alone. This shows that minimal muscle activity is needed to swing the shank during walking at constant normal speeds. This ballistic motion would have to be controlled by much greater muscle activity at other speeds. This explains why energy consumption is so low at "normal" walking speeds.

Figures 3-8 show the successive positions of the prosthesis in swing-phase. These stick diagrams give the impression that the amputee can trip in mid-swing. This is because: (a) The hip trajectory of a "normal" person has been used. Amputees, presumably raise their hips slightly for the prosthesis to clear the ground surface. (b) In the case of normal persons there is movement at the ankle which allows them to clear the ground. This has not been simulated as our interest is mainly in the role of the free swing of the shank.

The results show that for a given mass, geometry and location of centre of mass of the shank the walking speed cannot be varied very much. Within allowable limits of these parameters, "normal" gait can only be obtained if walking speeds are kept around 1.1 m/s to 1.2 m/s. Results are shown in Figures 3 to 8.

Clinical observations

Over 200 such limbs have been followed up for over three years and while problems are being faced with regard to the durability of the knee joints, the amputees seem to be pleased not only because they can squat but because the limb is much lighter. Many amputees who were earlier using the exoskeletal limb were provided with the endoskeletal version and most of them preferred the latter.

The alignment stability provided by the offset knee joint makes them feel so secure that most of them do not require to lock the knee when walking on a level surface. There is observed a remarkably improved swing-phase after a little practice. The stride length and speed of walking
is adjusted to an optimum level to provide a near-natural gait. Altered speeds of walking do pose problems. Since there is no friction at the knee, the leg continues to swing till it abruptly comes to a halt with an audible snap. The temptation to add friction etc. was resisted in order to retain a basic simplicity and a low cost. These are important considerations in low income countries.

Discussion

It is slowly being recognised that poor amputees from rural areas in India do not avail themselves of rehabilitation services unless the period of stay at a limb fitting centre is of small duration and the prosthesis provided does not need frequent repairs. The AK prosthesis described above is made from locally available materials and can be fitted in a relatively short time.

The results above indicate that at average walking speeds it is possible to optimise the swing-phase of the shank by an appropriate location of the centre of mass and by controlling the timing of the toe-off so that the shank can swing into the right position for heel-strike. The finding that the timing and inclination of the footpiece at the toe-off phase of the prosthesis largely determines the swing-phase characteristic is interesting. Most amputees seem to learn this after a variable period by trial and error, but this observation can be built into the gait training programme with considerable advantage.

If the mass of the footpiece is reduced then it would be easier to shift the location of the centre of mass by shifting a weight along the shank thus allowing for alterations in walking speed. Therefore it should be feasible to design a simpler alternative to the use of more
complicated and therefore more expensive knee mechanisms. Further studies seem to be called for to exploit this finding so that an AK prosthesis more suited for the rural populations of India can be developed.

REFERENCES


